

# ANALYSIS OF UNBOUNDED AND BOUNDED CIRCUITS AND ANTENNAS CONSIDERING FINITE EXTENT AND INHOMOGENEOUS DIELECTRIC

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## ABSTRACT

A field-theoretical algorithm is presented for characterizing unbounded and bounded circuits and antennas. Finite extent and inhomogeneous dielectric layer are rigorously considered in this method of lines-based model. The unbounded effects are determined with an improved lossy absorbing boundary condition (LABC) which can handle both propagating and evanescent waves. This analysis accounts for all the physical effects including electromagnetic coupling, evanescent higher-order modes, space-wave radiation and surface-wave leakage losses. Examples are given for unbounded loss effects including microstrip open-end deposited on a finite dielectric substrate and gap discontinuities on an inhomogeneous layer. Results indicate that the unbounded loss may be controlled by certain finite extent of the dielectric layer.

## INTRODUCTION

Planar integrated structures such as microstrip components have widely been used in microwave and millimeter-wave ICs and antennas when unbounded topology is considered. Accurate analysis and modeling of microstrip discontinuities constitute the cornerstone of the circuit performance prediction leading to effective CAD procedures. Passive M(H)MICs may be classified into bounded and unbounded structures. Bounded microstrip discontinuities such as steps, open-ends, gaps and stubs have been investigated intensively by using the method of

lines with hybrid homogeneous or inhomogeneous boundary conditions [3]-[5] while unbounded microstrip structures were mainly modeled by use of the method of moments [6]-[9] in the case of considering infinitely extended homogeneous substrate. If the dielectric substrate is finitely extended, inhomogeneous substrate used to suppress surface wave, to name an application example, the method of moments may not be applicable in this case.

The method of lines has been well established as a versatile numerical tool which provides a simple means to deal with complex structures, especially multilayered planar structures. This method has been successfully used in the analysis of shielded microstrip discontinuities on homogeneous substrates, while it has limited applications in the characterization of unbounded microstrip discontinuity problems because of the lack of an efficient absorbing boundary condition (ABC).

Since unbounded microstrip discontinuities are subject to the existence of surface wave modes, accurate high-frequency microstrip design requires the understanding of all the unbounded loss effects. Therefore, it is essential that appropriate boundary conditions enforced on the terminating walls absorb both the propagating and evanescent waves. An early proposed second-order approximate ABC [1] was efficient in the modeling of two-dimensional problems and three-dimensional resonant problems where evanescent waves can be ignored. However in the modeling of a discontinuity problem, it is imperative to account for these unbounded effects in the ABC.

This work uses an improved lossy absorbing boundary condition (LABC) [2] in which an artificial lossy factor was introduced in the ABC for the analysis of open microstrip discontinuities. The effects of finite and/or inhomogeneous substrate are rigorously considered.

## THEORY

For a multilayered structure, electromagnetic fields in each layer of the dielectric can be described by two vector potential functions  $\Pi^e$  and  $\Pi^h$ , which have only one component  $\Pi^e$  and  $\Pi^h$  in the z- direction if the layer is homogeneous or inhomogeneous along z-direction, which is also denoted as the propagating direction of the waves.  $\Pi^e$  and  $\Pi^h$  satisfy the Helmholtz equation when applied to describe the homogeneous medium. As for inhomogeneous dielectric where a space-dependence of the permittivity  $\epsilon_r(z)$  is assumed, the scalar potentials  $\Pi^h$  and  $\Pi^e$  must satisfy the Helmholtz and the Sturm-Liouville differential equations respectively.

The potential functions  $\Pi^e$  and  $\Pi^h$  are discretized in the x- and z- directions (c.f. Fig.1) with a nonequidistant scheme. The continuity conditions for the tangential electric and magnetic field components must be fulfilled between different dielectric media. The effect of the finite and/or inhomogeneous substrate can be taken into account by properly filling in the matrix of  $r^{e(h)}(z)$

$$r^{e(h)}(z) \rightarrow \text{diag}(r^{e(h)}(z_{e(h)})) \quad (1)$$

On the side terminating walls, an improved LABC is applied in which a nonphysical lossy layer is defined [2] with a thickness of  $h_x$  and a complex term  $r = r' - j r''$  to absorb the propagating and evanescent waves. The incident and reflected waves at the input port are modeled on the basis of the source approach [5] which leads to inhomogeneous Dirichlet and Neumann boundary conditions imposed respectively on  $\Pi^e$  and  $\Pi^h$  at  $z = 0$ . In the case that the output terminals are matched, we have

$$\left. \Pi^e \right|_{z=0} = (1 - S_{11}) \Pi^e_0 \quad (2)$$

$$\left. \frac{\partial \Pi^h}{\partial z} \right|_{z=0} = -j Y_1 (1 - S_{11}) \Pi^h_0$$

(3)

where  $\Pi^e_0$ ,  $\Pi^h_0$  and  $Y_1$  are derived from the uniform transmission line problem.  $S_{11}$  is the reflection coefficient at the input port when the output terminals are matched.

For a one-port circuit, the LABC is directly applied on the output terminating wall in z- direction. For a two-port circuit of which the output reference port is located at a distance far enough from the discontinuities, and only propagating waves are assumed to exist with the propagating constant  $\beta_z$ , the boundary condition is formulated as follows in considering magnetic lines on the terminating wall in z- direction

$$\left. \frac{\partial \Pi^h}{\partial z} \right|_{z=N_z} = -A_z \Pi^h_{N_z}$$

(4)

where

$$A_z = j Y_2 e^{-j \beta_z \frac{e_{zn}}{2}} \quad (5)$$

The matrix characterizing the normalized nonequidistant scheme becomes

$$[r_{ze}] = \begin{bmatrix} \sqrt{h_z/e_{z1}} & & & \\ & \sqrt{h_z/e_{z2}} & & \\ & & \ddots & \\ & & & \sqrt{A_z e_{zN_z}} \sqrt{h_z/e_{zN_z}} \end{bmatrix}$$

(6)

and  $S_{21}$  is defined as follows

$$S_{21} = \frac{J_2}{J_1^+} \sqrt{\frac{Z_2}{Z_1}} \quad (7)$$

where  $J_1^+$  is the incident current density at the input port 1, and  $J_2$  is the outgoing current density at the output port 2.  $Z_1$  and  $Z_2$  are characteristic impedances of the transmission lines at port 1 and port 2.

## RESULTS AND DISCUSSIONS

The LABC has been successfully used in the 2-D uniform transmission line problem [2]. To verify the effectiveness of the LABC in its application to unbounded microstrip discontinuity problems, we have calculated the S parameters of microstrip open-ends with (in)finite substrate and microstrip gap discontinuities with (in)homogeneous substrates, which are useful in the design of matching stubs and coupled-line filters.

Fig.1 shows the generalized geometry of a microstrip open-end and a gap discontinuity with a line width  $w$ . The dielectric substrate having height  $h$  and relative dielectric constant  $\epsilon_r$  can be of infinite or finite extent, and may also be inhomogeneous in  $z$ -direction. The LABC is first used in the characterization of a microstrip open-end deposited on an infinite substrate. Fig.2 shows the frequency-dependent reflection coefficient of such a structure. Comparison of the reflection coefficient with the results obtained by the method of moments [8] is also made, showing a satisfactory agreement. Obviously, increased power loss with frequency is found for such a radiating structure. In practical situation, the finite substrate may excite complicated surface waves. Effects of the finite substrate on the reflection coefficient are investigated, and the results are shown in Fig.3, where the ratio of the finite extent of the substrate over the line width is changed from 1 to 4, and the height of the substrate is 15 mil and 25 mil respectively. It can be seen that the reflection coefficient is influenced dramatically by the size of the substrate. The results indicate that high power loss may take place in a certain range of this ratio close to 1.2, and a thicker substrate may cause a higher power loss in general.

Microstrip gap discontinuity presents a two-port example as shown in Fig. 4. A simple discontinuity on homogeneous substrate is first characterized by using the proposed boundary condition. The transmission and reflection coefficients are obtained and compared to the published experimental data [10], indicating a very good agreement. It is known that a gap discontinuity may excite surface waves, thus leading to power loss. One way to suppress such a surface wave loss is to remove part of the substrate (in the form of a slot) between the two microstrip

lines as shown in Fig.1b. It can be expected that the resulting slot width affects the behavior of the surface wave. Fig.5 examines the power loss versus the slot width, suggesting that a minimum loss may be achieved if the  $t/w$  is selected around 0.3 for  $h=25$  mil, and 0.05 for  $h=15$  mil. Therefore, it can be concluded that the surface wave can be effectively suppressed by a proper design of the inhomogeneous dielectric layout.

## CONCLUSIONS

The use of method of lines with an improved lossy absorbing boundary condition (LABC) leads to a generalized efficient algorithm that can be used to predict unbounded and bounded high-frequency ICs and antennas. Realistic topologies can be accurately modeled such as finite and/or inhomogeneous dielectric substrate encountered in the design of microstrip open-ends and microstrip gap discontinuities. Examples presented in this paper indicate that the unbounded power loss may be suppressed or reduced through an appropriate design of the substrate layout. This further suggests that field-theoretical analysis and CAD are useful in the power loss prevention and high-quality circuit design.

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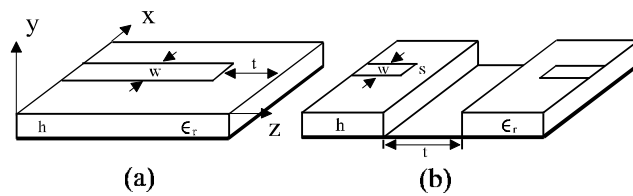


Figure 1: Geometry of Microstrip Open-ends and Gap Discontinuities

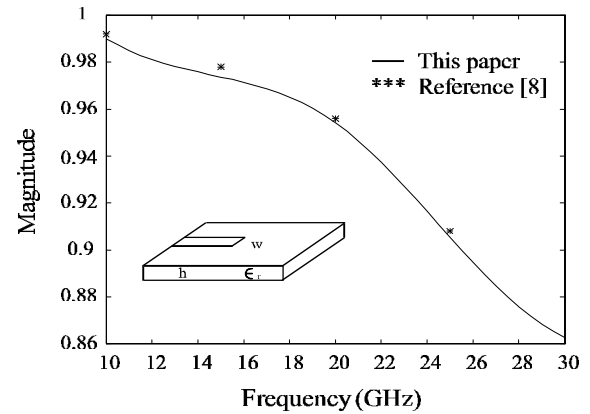


Figure 2: Reflection Coefficient of Microstrip Open-ends ( $\epsilon_r = 9.9, w = h = 25\text{mil}$ )

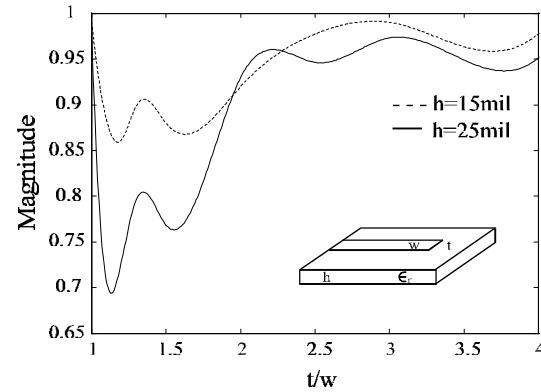


Figure 3: Reflection Coefficient of Microstrip Open-ends with Finite Extent ( $\epsilon_r = 9.9, w = 25\text{mil}, f = 20\text{GHz}$ )

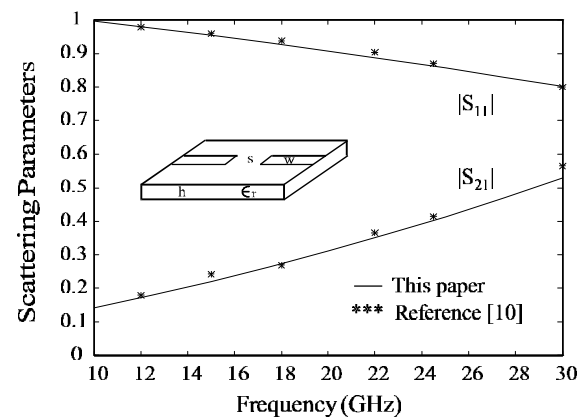


Figure 4: S-parameters of Microstrip Gap Discontinuities ( $\epsilon_r = 9.9, w = h = 25\text{mil}, s = 0.35w$ )

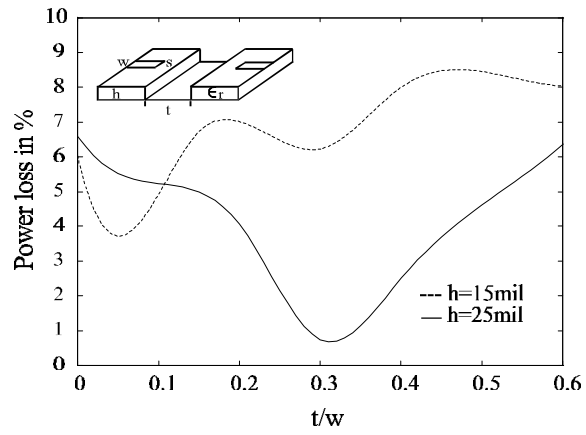


Figure 5: Power Loss versus the Width of the Slot  
for Microstrip Gap Discontinuities  
( $\epsilon_r = 9.9, w = 25\text{mil}, s = 0.175w, f = 30\text{GHz}$ )